Numerical simulation of the discrete equilibrium solutions of Boltzmann equation on a *N*-layer loosely coupled hexagonal grid

Wahida Zaman Loskor

Abstract- We perform numerical simulation of the discrete equilibrium solutions (equilibria) of Boltzmann equation on a *N*-layer loosely coupled hexagonal grid. The equilibria f of the Boltzmann equation can be expressed in terms of four parameters characterizing mass, (x, y)-momenta and kinetic energy. The computation of discrete equilibria of the Boltzmann equation is effected by varying temperature and bulk-velocity. The temperature depends on the parameter μ which is characterizing kinetic energy. The bulk-velocity depends on the parameter $\kappa's$ which are characterizing (x, y) momenta and μ as well.

Keywords— The Boltzmann equation, numerical simulation, loosely coupled hexagonal grid, discrete equilibrium solution, bulk-velocity, kinetic energy, momenta.

1 Introduction

In recent years many researchers have proposed several numerical techniques to deal with the complexity of the Boltzmann collision operator. Simplified collision model have been introduced in rectangular grid in [H. Babovsky (2001); H. Babovsky (2002)], but there appears many artificial invariants which have to be eliminated by further techniques. Based on hexagonal grid in R^2, [Andallah and Babosky (2003)] has introduced simplified collision model [as a discrete velocity model by T. Platkowski and R. Illner (1988)] and it has also established a kinetic theory for the discrete Boltzmann equation in which discrete equilibrium solutions (equilibria) of the Boltzmann equation is presented in terms of four parameters characterizing mass, momenta and kinetic energy. Later on [Andallah (2004)] has developed an automatic generation of the Boltzmann collision operator based on any larger size haxagonal grid and made some numerical simulation based on the grid in R^2. [Andallah and Babovsky (2005)] has introduced a discrete model Boltzmann equation based on a loosely coupled hexagonal discretization of R^2. This model satisfies the basic features of kinetic theory like conservation laws, H-theorem; correct dimension of the null-space of the linearized collision operator etc. [Andallah, Babovsky and Khan (2006)] has developed the existence of two classes of regular hexagons in a loosely coupled hexagonal grid in R^2. It is also noted that the model of the Boltzmann equation, based on only binary collision law, descretized on the loosely hexagonal grid in R^2, provides two spurious invariants. [Andallah (2008)] has developed a generalized layer-wise construction of a loosely coupled N-layer hexagonal mesh for a discrete model Boltzmann equation. The work also described some properties of the N-layer loosely coupled hexagonal grid and identify the regular hexagons belonging to the mash in order to generate collision model for the Boltzmann equation. The discrete equilibrium solutions (equilibria) of the Boltzmann equation discretized on a generalize N-layer loosely coupled hexagonal grid is determined in [W. Z. Loskor and Andallah (2014)]. The equilibria f of the discrete Boltzmann equation can be expressed in terms of four parameters characterizing mass, (x, y)-momenta and kinetic energy which are established by induction hypothesis.

This research article performs numerical simulation of the discrete equilibrium solutions (equilibria) of Boltzmann equation on a *N*-layer loosely coupled hexagonal grid. It is also observe that the computation of discrete equilibria of the Boltzmann equation is effected by varying temperature and bulk-velocity. Besides these, this article produce numerical results showing how temperature depends on the parameter μ, characterizing kinetic energy and bulk-velocity depends on the parameter $\kappa's$, characterizing (x, y) momenta and μ as well.

2 Boltzmann Equation

The Boltzmann equation is a prominent representative of kinetic equations, describes the evolution of rarefied gases. The dynamics of the Boltzmann equation is given by a free flow step and a particle interactions step with the conservation of momentum and energy. The free flow step is modeled by the Liouville-equation and the particle interaction step is modeled by the Boltzmann collision operator. As a simple mathematical consequence of the Boltzmann equation is $\left(\partial_t + v.\nabla_x\right) f\left(t, x, v\right) = \mathrm{J[f, f]}$, where $f = f(t,x,v)$, a density

- *Faculty of Science & Humanities, Bangladesh Army International University of Science & Technology (BAIUST), Comilla Cantonment, Comilla, Bangladesh, PH:+8801711247965, E-mail: loskor.baiust@gmail.com, loskor@baiust.edu.bd .*

function which depends on time, space and velocity and $J[f,f] = \int_{R^d} \int_{S^{d-1}} k(v-w,\eta)[f(v')\,f(w') - f(v)\,f(w)]d^{d-1}\eta \; d^d w$ is the Boltzmann collision operator.

3 *N*-LAYER LOOSELY COUPLED HEXAGONAL MESH

In this section, a generalized layer-wise construction of a loosely coupled *N*-layer hexagonal mesh for a discrete velocity model Boltzmann equation is presented. Fig.1.1 shows a 72-velocity model constructed by adding two-layer of hexagons centering to a center one and called two layers loosely coupled hexagonal mesh. Similarly by adding one more layer of regular basic hexagons, one can obtain a 3-layer hexagonal mesh and so on. In general we may call this a *N*-layer loosely coupled hexagonal mesh and the collision model based on the mesh can be called a *N*-layer hexagonal model which is a regular collision model so that it satisfies the basic kinetic features and can be divided into six symmetric partitions (Fig.1).

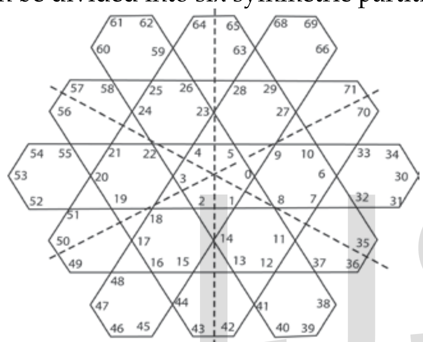


Fig. 1: 72-velocity Model as a Two-Layer Model.

4 EQUILIBRIA FOR A N -LAYER MODEL

Strictly positive density vectors $f = \left(f_i\right)_{i=0}^{(3N+2)^2 + (3N+1)}$ is said to be the equilibrium solutions (equilibria) if $J[f,f] \equiv 0$ for a N -layer hexagonal model. The i^{th} equilibrium of the n^{th} layer is $f(n,i) = z\mu^{m(n,i)}\,\kappa_{0+}^{\bar{\kappa}_0(n,i)}\,\kappa_{1+}^{\bar{\kappa}_1(n,i)}\,\kappa_{2+}^{\bar{\kappa}_2(n,i)}$; where $z, \kappa_{0+}, \kappa_{1+}, \kappa_{2+} > 0$ are satisfying arbitrary quantities $\kappa_{0+}\kappa_{1-}\kappa_{2+} = 1$.

The equilibria at the six nodes of 0-st layer (i.e. at the nodes of the central basic hexagon) are given by $\left(f_0, f_1, f_2, f_3, f_4, f_5\right) = z.\left(\kappa_{0+}, \kappa_{1+}, \kappa_{2+}, \kappa_{0-}, \kappa_{1-}, \kappa_{2-}\right)^{T}$.

For a 3-layer model, presents the equilibria for the nodes of the partition as

$$\left.\begin{array}{l} z(\mu\,\kappa_{0+}\kappa_{1+},\;\; \mu^3\kappa_{0+}\kappa_{1+}^2,\;\; \mu^4\kappa_{1+}^3,\;\; \mu^3\kappa_{1+}^2\kappa_{2+}) \in \text{1st layer} \\ z(\mu^9\,\kappa_{0+}^2\kappa_{1+}^3,\;\; \mu^6\kappa_{0+}\kappa_{1+}^3,\;\; \mu^{10}\kappa_{0+}\kappa_{1+}^4,\; \mu^{12}\kappa_{1+}^5, \\ \quad \mu^{10}\kappa_{1+}^4\kappa_{2+},\;\; \mu^6\kappa_{1+}^3\kappa_{2+},\;\; \mu^9\kappa_{1+}^3\kappa_{2+}^2) \in \text{2nd layer} \\ z(\mu^{13}\,\kappa_{0+}^3\kappa_{1+}^3, \mu^{18}\kappa_{0+}^3\kappa_{1+}^4, \mu^{19}\kappa_{0+}^2\kappa_{1+}^5, \mu^{15}\kappa_{0+}\kappa_{1+}^5, \mu^{21}\kappa_{0+}\kappa_{1+}^6, \\ \quad \mu^{24}\kappa_{1+}^7, \mu^{21}\kappa_{1+}^6\kappa_{2+}, \mu^{15}\kappa_{1+}^5\kappa_{2+}, \mu^{19}\kappa_{1+}^5\kappa_{2+}^2, \mu^{18}\kappa_{1+}^4\kappa_{2+}^3) \in \text{3rd layer} \end{array}\right| \quad (1.1)$$

Where z parameterizes mass, $(\kappa_{0+}, \kappa_{2+})$ characterize non-vanishing bulk-velocity, μ responsible kinetic energy.

Fig. 2 shows each n^{th} layer of a partition has $(3n+1)$ nodes and the node numbering is from the top to bottom of at each layer. These values of equilibria for a partition of a N -layer model are generalize as in the proposition below.

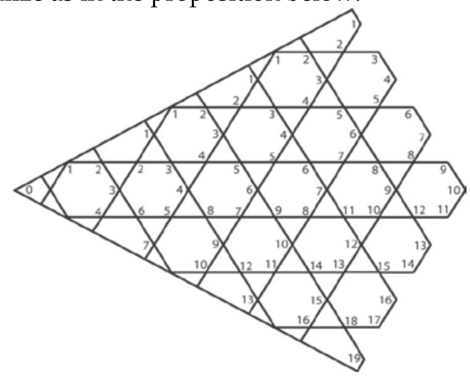


Fig. 2: A partition of a Six-Layer Model.

4.1 Proposition

For a partition (of a N -layer model) corresponding to the triple $z.(\kappa_{0+}, \kappa_{1+}, \kappa_{2+})$, the equilibria is described in-terms of the parameters $z, \kappa_{0+}, \kappa_{1+}, \kappa_{2+}$ as in the following steps.

In the case of odd layer:

(a) For kinetic energy μ :

Corresponding to the values of $(3m+1)^{\text{th}}$ $m = 0, \cdots, \frac{n-1}{2}$; $(3m+2)^{\text{th}}$ $m = 0, \cdots, \frac{n-3}{2}$, and $(3m+3)^{\text{th}}$ $m = 0, \cdots, \frac{n-3}{2}$ nodes of n^{th} layer, they obtained values by the $(3m+1)^{\text{th}}$, $(3m+2)^{\text{th}}$, and $(3m+3)^{\text{th}}$ nodes of the $(n-2)^{\text{th}}$ layer with an increment μ^{6n-6}, μ^{6n-3}, and μ^{6n-3} respectively. The value of $\left(\frac{3n+1}{2}\right)^{\text{th}}$ node of n^{th} layer, it obtained value by the $\left(\frac{3n-1}{2}\right)^{\text{th}}$ node of n^{th} layer with an increment μ^{2n} . The value of $\left(\frac{3n-1}{2}\right)^{\text{th}}$ node of n^{th} layer, it obtained value by the $\left(\frac{3n-1}{2}\right)^{\text{th}}$ node of n^{th} layer with an increment μ^{n} . The rest values of $\left(i = \frac{3n+5}{2}\right)^{\text{th}}$ node to $\left(i = 3n+1\right)^{\text{th}}$ node of n^{th} layer, they obtained values by the $\left(3n+3-i\right)^{\text{th}}$ node of n^{th} layer respectively.

(b) For non-vanishing bulk velocity κ_{0+} :

Corresponding to the values of 1st node to 3rd node of n^{th} layer, they obtained values by the 1st node to 3rd node of the

$(n-2)^{th}$ layer with an increment κ_{0+}^{2}. The value of $(i=4)^{th}$ node to $\left(i=\frac{3n+3}{2}\right)^{th}$ node of n^{th} layer, they obtained value by the $(i-3)^{th}$ node of n^{th} layer with decrease κ_{0+}^{-2}. The rest values of $\left(\frac{3n+5}{2}\right)^{th}$ node to $(3n+1)^{th}$ node of n^{th} layer, their values are κ_{0+}^{0}.

(c) For non-vanishing bulk velocity κ_{1+}:

Corresponding to the values of 1st node to 3rd node of n^{th} layer, they obtained values by the 1st node to 3rd node of the layer with an increment κ_{1+}^{2}. The value of $(i=4)^{th}$ node to $\left(i=\frac{3n+3}{2}\right)^{th}$ node of n^{th} layer, they obtained value by the $(i-3)^{th}$ node of n^{th} layer with an increment κ_{1+}^{2}. The rest values of $\left(\frac{3n+5}{2}\right)^{th}$ node to $(3n+1)^{th}$ node of n^{th} layer, they obtained values by the $(3n+3-i)^{th}$ node of n^{th} layer respectively.

(d) For non-vanishing bulk velocity κ_{2+}:

The values of 1st node to $\left(\frac{3n+3}{2}\right)^{th}$ node of n^{th} layer, their values are κ_{2+}^{0}. The values of $\left(\frac{3n+5}{2}\right)^{th}$ node to $\left(\frac{3n+7}{2}\right)^{th}$ node of n^{th} layer, their values are κ_{2+}^{1}. The rest values of $\left(i=\frac{3n+9}{2}\right)^{th}$ node to $(i=3n+1)^{th}$ node of n^{th} layer, they obtained values by the $(i-3)^{th}$ node of n^{th} layer with an increment κ_{2+}^{2}.

In the case of even layer:

(a) For kinetic energy μ:

Corresponding to the values of $(3m+1)^{th}$ $m=0,\cdots,\frac{n-2}{2}$, $(3m+2)^{th}$ $m=0,\cdots,\frac{n-2}{2}$, and $(3m+3)^{th}$ $m=0,\cdots,\frac{n-4}{2}$ nodes of n^{th} layer, they obtained values by the $(3m+1)^{th}$, $(3m+2)^{th}$, and $(3m+3)^{th}$ nodes of the $(n-2)^{th}$ layer with an increment μ^{6n-3}, μ^{6n-6}, and μ^{6n-3} respectively. The value of $\left(\frac{3n}{2}\right)^{th}$ node of n^{th} layer, it obtained value by the $\left(\frac{3n-2}{2}\right)^{th}$ node of n^{th} layer with an increment μ^{2n}. The value of $\left(\frac{3n+2}{2}\right)^{th}$ node of n^{th} layer, it obtained value by the $\left(\frac{3n}{2}\right)^{th}$ node of n^{th} layer with an increment μ^{n}. The rest values of $\left(i=\frac{3n+4}{2}\right)^{th}$ node to $(i=3n+1)^{th}$ node of n^{th} layer, they obtained values by the $(3n+2-i)^{th}$ node of n^{th} layer respectively.

(b) For non-vanishing bulk velocity κ_{0+}:

Corresponding to the values of 1st node to 3rd node of n^{th} layer, they obtained values by the 1st node to 3rd node of the $(n-2)^{th}$ layer with an increment κ_{0+}^{2}. The value of $(i=4)^{th}$ node to $\left(i=\frac{3n+2}{2}\right)^{th}$ node of n^{th} layer, they obtained value by the $(i-3)^{th}$ node of n^{th} layer with a decrease κ_{0+}^{-2}. The rest values of $\left(\frac{3n+4}{2}\right)^{th}$ node to $(3n+1)^{th}$ node of n^{th} layer, their values are κ_{0+}^{0}.

(c) For non-vanishing bulk velocity κ_{1+}:

Corresponding to the values of 1st node to 3rd node of n^{th} layer, they obtained values by the 1st node to 3rd node of the $(n-2)^{th}$ layer with an increment κ_{1+}^{2}. The value of $(i=4)^{th}$ node to $\left(i=\frac{3n+2}{2}\right)^{th}$ node of n^{th} layer, they obtained value by the $(i-3)^{th}$ node of n^{th} layer with an increment κ_{1+}^{2}. The rest values of $\left(i=\frac{3n+4}{2}\right)^{th}$ node to $(i=(3n+1))^{th}$ node of n^{th} layer, they obtained values by the $(3n+2-i)^{th}$ node of n^{th} layer respectively.

(d) For non-vanishing bulk velocity κ_{2+}:

The values of 1st node to $\left(\frac{3n+2}{2}\right)^{th}$ node of n^{th} layer, their values are κ_{2+}^{0}. The values of $\left(\frac{3n+4}{2}\right)^{th}$ node to $\left(\frac{3n+6}{2}\right)^{th}$ node of n^{th} layer, their values are κ_{2+}^{1}. The restvalues of $\left(i=\frac{3n+8}{2}\right)^{th}$ node to $(i=3n+1)^{th}$ node of n^{th} layer, they obtained values by the $(i-3)^{th}$ node of n^{th} layer with an increment κ_{2+}^{2}. The equilibria for other partitions as well as for the complete N -layer model is determine by symmetry.

Now if κ_{1+} is substitute by $\kappa_{0+}\kappa_{2+}$ in equation (1.1) then it has the following statement.

4.2 Theorem

Let N_0, N_e denote respectively the set of odd and even natural numbers. The i^{th} equilibria in the n^{th} layer of the partition corresponding to the triple $(\kappa_{0+}, \kappa_{1+}, \kappa_{2+})$ is given by

$$f(n,i) = z\,\mu^{m(n,i)}\,\kappa_{0+}^{\bar\kappa_0(n,i)}\,\kappa_{2+}^{\bar\kappa_2(n,i)};\ n = 0,\cdots,N \text{ and } i = 1,\cdots,3n+1 \quad (1.2)$$

In the case of $n \in N_0$ and

For $i = 1,\cdots,\frac{3n+3}{2}$; $k = 0,\cdots,\frac{n-1}{2}$

$$m(n,i) = \frac{3n^2-1}{2} + d_i, \quad d_{i=3k+1} = 2k^2,$$
$$= \frac{3n^2+3n}{2} + d_i, \quad d_{i=3k+2} = 2k^2 + k,$$
$$= \frac{3n^2+3n+2}{2} + d_i, \quad d_{i=3k+3} = 2k^2 + 3k.$$
$$\bar\kappa_0(n,i) = 2n + d_i, \quad d_{i=3k+1} = 0,$$
$$= 2n+1+d_i, \quad d_{i=3k+2} = 0,$$
$$= 2n+1+d_i, \quad d_{i=3k+2} = 0.$$
$$\bar\kappa_2(n,i) = n + d_i, \quad d_{i=3k+1} = 2k,$$
$$= n+1+d_i, \quad d_{i=3k+2} = 2k,$$
$$= n+2+d_i, \quad d_{i=3k+3} = 2k.$$

For rest $i = \frac{3n+5}{2},\cdots,3n+1$,

$$m\left(n, i_{(=\frac{3n+5}{2},\cdots,3n+1)}\right) = m\left(n, i_{(=\frac{3n+1}{2},\cdots,2)}\right),$$
$$\bar\kappa_0\left(n, i_{(=\frac{3n+5}{2},\cdots,3n+1)}\right) = \bar\kappa_2\left(n, i_{(=\frac{3n+1}{2},\cdots,2)}\right),$$
$$\bar\kappa_2\left(n, i_{(=\frac{3n+5}{2},\cdots,3n+1)}\right) = \bar\kappa_0\left(n, i_{(=\frac{3n+1}{2},\cdots,2)}\right).$$

In the case of $n \in N_e$ and

For $i = 1,\cdots,\frac{3n}{2}$; $k = 0,\cdots,\frac{n-2}{2}$

$$m(n,i) = \frac{3n^2+3n}{2} + d_i, \quad d_{i=3k+1} = 2k^2 + 2k,$$
$$= \frac{3n^2}{2} + d_i, \quad d_{i=3k+2} = 2k^2 + 2k,$$
$$= \frac{3n^2+3n+2}{2} + d_i, \quad d_{i=3k+3} = 2k^2 + 3k.$$
$$\bar\kappa_0(n,i) = 2n+1+d_i, \quad d_{i=3k+1} = 0,$$
$$= 2n + d_i, \quad d_{i=3k+2} = 0,$$
$$= 2n+1+d_i, \quad d_{i=3k+2} = 0.$$
$$\bar\kappa_2(n,i) = n+1+d_i, \quad d_{i=3k+1} = 2k,$$
$$= n+1+d_i, \quad d_{i=3k+2} = 2k,$$
$$= n+2+d_i, \quad d_{i=3k+3} = 2k.$$

For $i = \frac{3n+2}{2}$, $m(n,i) = 2n^2 + 2n$;

$\bar\kappa_0(n,i) = 2n+1$; $\bar\kappa_2(n,i) = 2n+1$

For rest $i = \frac{3n+4}{2},\cdots,3n+1$,

$$m\left(n, i_{(=\frac{3n+4}{2},\cdots,3n+1)}\right) = m\left(n, i_{(=\frac{3n}{2},\cdots,1)}\right),$$
$$\bar\kappa_0\left(n, i_{(=\frac{3n+4}{2},\cdots,3n+1)}\right) = \bar\kappa_2\left(n, i_{(=\frac{3n}{2},\cdots,1)}\right),$$
$$\bar\kappa_2\left(n, i_{(=\frac{3n+4}{2},\cdots,3n+1)}\right) = \bar\kappa_0\left(n, i_{(=\frac{3n}{2},\cdots,1)}\right).$$

Proof: The proof of the theorem is at 5.2 of [Loskor (2012)].

5 Computation of Equilibrium

Here the discrete equilibria are computed, which are described by the parameters $z, \mu, \kappa_{0+}, \kappa_{2+}$ characterizing respectively mass, temperature, and bulk-velocity. It is evident that $\bar v_x = 0, <0, >0$ according as $\kappa_{0+}\kappa_{2+} = \kappa_{1+} = 1, <1, >1$; $\bar v_y = 0, <0, >0$ according as $\kappa_{0+} - \kappa_{2+} = 0, <0, >0$ and

$$\kappa_{0+} - \kappa_{2+} = \kappa_{0+} - \frac{\kappa_{1+}}{\kappa_{0+}}. \quad (1.3)$$

Now the discrete equlibria $\tilde f^h \in \varepsilon$ given by the previous theorem for the case of zero bulk-velocity on a 4-layer grid (of 210 grid points) with discretization parameter $h=1$ for three different value of $\mu = 0.25,\ 0.55,\ 0.95$ is computed and the corresponding Maxwellian given by $\tilde f = \frac{\rho}{(2\pi T)^{d/2}}\exp\left(\frac{-(v-\bar v)^2}{2T}\right)$, $d = 2$, where $\rho := \sum_i \tilde f_i^h$ is the mass density, $\bar v = \left(1/\rho\right)\sum_i v_i \tilde f_i^h$ is defined as the bulk velocity, $T = \left(1/2\rho\right)\sum_i (v_i - \bar v)^2 \tilde f_i^h$ is the temperature. The normalized discrete equilibrium state f^h is compared with the normalized Maxwellian f and calculated the error $err = \left\|f - f^h\right\|_1$ and the moments, temperature as shown in the table 1.1.

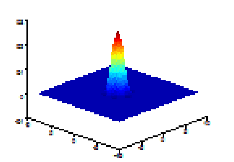
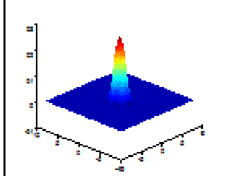
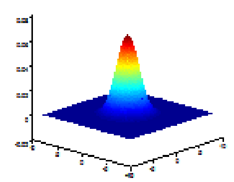
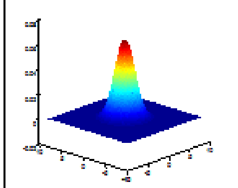
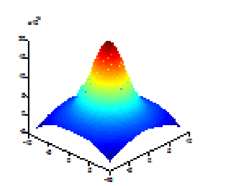
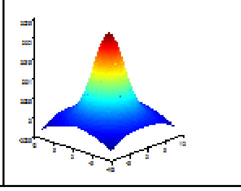
f^h = Discrete equilibria	f = Maxwellian	Err $= \|f - f^h\|_{L_1}$
		$\rho = 1.000$ $\bar{v}_x = 0$ $\bar{v}_y = 0$ $\mu = 0.25$ $T = 0.8293$ $err = 0.0501$
		$\rho = 1.000$ $\bar{v}_x = 0$ $\bar{v}_y = 0$ $\mu = 0.55$ $T = 1.6759$ $err = 0.0013$
		$\rho = 1.000$ $\bar{v}_x = 0$ $\bar{v}_y = 0$ $\mu = 0.95$ $T = 11.0488$ $err = 0.1920$

Table 1.1: In varying temperatures Discrete equilibria and Maxwellian are on a 4-layer grid.

Here three interesting cases of three different temperatures with zero bulk-velocities are presented.

1. In the first case, for $\mu = 0.25$ the calculated temperture is $T = 0.8293$. The main part of the configuration is centered at the origin with a small radius and a small part of the mass occurred on the grid. That is the resolution of the grid is too low to present such low temperature and this causes a noticeable 5% error.
2. This is a good situation because the main part of the mass of the function f lies inside of the domain. In this case it occurs very little error for $\mu = 0.55$ in which the calculated temperature $T = 1.6759$.
3. Here the grid is not large enough to present such high temperature $T = 11.0488$ for a given $\mu = 0.95$ and a significant fraction of the mass of the function f is cut down the boundary which causes 20% error.

Thus to avoid this error it requires to further extension of the 4-layer grid model. It is thus seen that a larger model is needed to restrict the error to a reasonable range for high temperature otherwise a noticeable error due to boundary effect.

Let us now observe the effect of varying bulk-velocity for a constant temperature. For this we choose $\mu = 0.3$ in a 6-layer hexagonal grid (of 420 grid points) and calculate the equilibrium solution for three different cases of bulk velocities $(v_x, v_y) = (0,0)$, $(0.3,0)$ and $(0.3, 0.5477)$. For these values of the parameters we compute the moments, temperature together with the heat flux components.

$$q_x^h = \frac{1}{2}\sum_i f_i.(v_{xi} - \bar{v}_x)((v_{xi} - \bar{v}_x)^2 + (v_{yi} - \bar{v}_y)^2)$$

$$q_y^h = \frac{1}{2}\sum_i f_i.(v_{yi} - \bar{v}_y)((v_{xi} - \bar{v}_x)^2 + (v_{yi} - \bar{v}_y)^2)$$

and the stress tensor components

$$p_{xx}^h = \sum_i f_i.(v_{xi} - \bar{v}_x)^2$$

$$p_{xy}^h = p_{yx}^h = \sum_i f_i.(v_{xi} - \bar{v}_x)(v_{yi} - \bar{v}_y)$$

$$p_{yy}^h = \sum_i f_i.(v_{yi} - \bar{v}_y)^2$$

The table 1.2 illustrate three different situations.

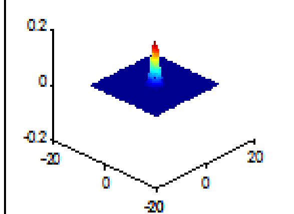
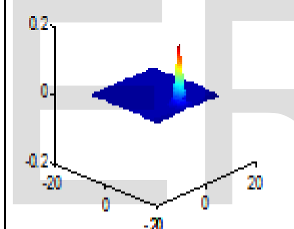
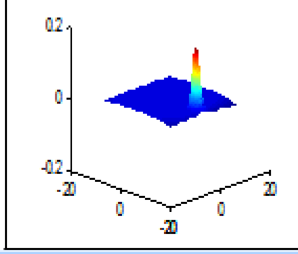
f^h = equilibra	m^h	p^h	q^h
	$\rho^h = 1.0000$ $u_x = 0$ $u_y = 0$ $T = 0.9753$ $err = 0.0163$	0.9753 0.0000 0.0000 0.9753	0.0000 0.0000
	$\rho^h = 1.0000$ $u_x = 9.6$ $u_y = 0$ $T = 0.9643$ $err = 0.0776$	0.9613 0.0000 0.0000 0.9672	-0.1518 0.0000
	$\rho^h = 1.0000$ $u_x = 9.7$ $u_y = 0$ $T = 0.9630$ $err = 0.0679$	0.9599 0.0074 0.0074 0.9661	-0.0333 -0.1766

Table 1.2: Discrete equilibria in varying bulk-velocity on a 6-layer grid

Here three interesting cases of three different bulk-velocities are presented.

1. In the first case both the components of bulk-velocity are zero and $f^h \in \varepsilon$ is symmetric about both the axes. In the stress tensor, both the principal diagonal elements are equal to the calculated temperature $T = 0.9753$ and both the off diagonal elements are zero. Here we observe vanishing heat flux components. As the main part of the mass lies inside the domain we obtain a very little error in this case.
2. In the second case, we impose positive x-component bulk-velocity and zero y-component bulk-velocity. In this

situation $f^h \in \varepsilon$ loses its symmetry property about the x-axis and we obtain non-zero x-component heat flux. However, the remaining symmetry property in the direction of $\bar{v}_y$ still guarantees the diagonal form of the stress tensor p^h. It is seen that the principal diagonal elements of the stress tensor are not equal anymore. On the other hand, temperature remains almost the same as in the first case which means nonzero bulk-velocity dose not influence the temperature. In this case we have more error than the first case which is due to a little boundary effect and this cause a little change of the temperature.

3. In the third case, both the components of the bulk-velocity are positive and this causes the non symmetric nature of $f^h \in \varepsilon$ about both the co-ordinate axes of the grid. The diagonal form of the stress tensor p^h is violated and both the components of q^h are negative. The temperature changes are little which is consistent with the little error due the boundary effect.

In all the three cases the main part of the configuration is symmetric about the centers of three different basic hexagon of the grid.

Now, Fig.1.3, shows the calculated temperature for corresponding to values of $\mu \in [0.1,\ 0.9]$.

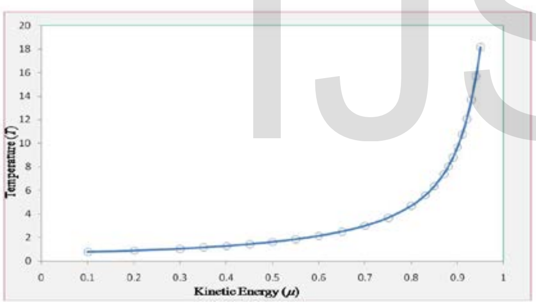


Fig: 1.3 Temperature T depends on Kinetic Energy μ**.**

It is clearly seen in the Fig.1.3 that temperature depends upon the values of the kinetic energy μ**.** which is expected.

Fig.1.4 shows the calculated bulk-velocity for given $\mu \in [0.1,\ 0.9]$ at the three different choice of $(\kappa_{0+}, \kappa_{2+})$.

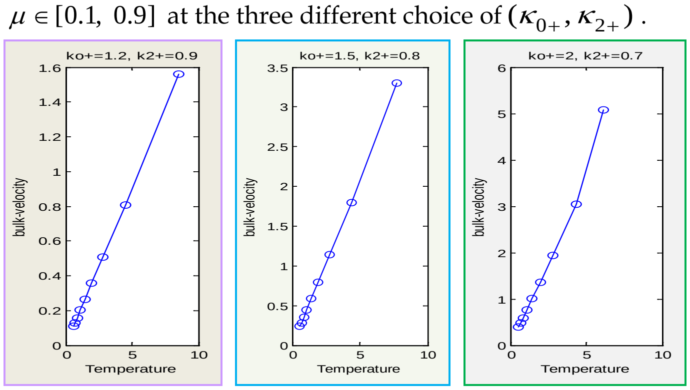


Fig: 1.4 Bulk-velocity depends on $T(\mu)$ and κ's

As expected, it is clearly seen in the Fig.1.4 that the modulus of the bulk-velocity $|v|$ depends upon the choice of temperature as well as the values of the parameters κ_{0+}, κ_{2+}.

6 Conclusion

The equilibria f of the Boltzmann equation can be expressed in terms of four parameters characterizing mass, (x, y)-momenta and kinetic energy. The computation of discrete equilibria of the Boltzmann equation is effected by varying temperature and bulk-velocity. The temperature depends on the parameter μ, characterizing kinetic energy. The bulk-velocity depends on the parameter κ's, characterizing (x, y) momenta and μ as well.

The discrete equilibria are useful for the error estimation of the discrete model Boltzmann equation. Restricting the error in a given tolerance one can investigates efficient numerical scheme to solve the space inhomogeneous Boltzmann equation which we may investigate in our future work.

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